

Multinational Production, Risk Sharing, and Home Equity Bias

Technical Appendix

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1 Model Details

This Appendix shows derivations for Section 2.

1.1 Derivation of price indices, demand for goods, and real exchange rate

First, we derive the price index in the home country, P_t . It consists of the price index of goods produced by home firms in the home country, P_{Ht} , and price index of goods produced by foreign firms in the home country, P_{Ft} . C_t is the home consumer's consumption basket

Now, we derive the price index of goods produced by home firms in the home country, P_{Ht} .

In this derivation, the home consumer's consumption basket, C_{Ht} , consists of goods produced by the home firms z where we integrate from 0 to a because there are a home firms:

$$\min p_t(z)c_t(z) \text{ subject to } C_{Ht} = 1 \text{ where } C_{Ht} = \left[\left(\frac{1}{a} \right)^{\frac{1}{\sigma}} \int_0^a c_t(z)^{\frac{1}{\sigma}} dz \right]^{\sigma}$$

$$L = p_t(z)c_t(z) - P_{Ht} \left[\left(\frac{1}{a} \right)^{\frac{1}{\sigma}} \int_0^a c_t(z)^{\frac{1}{\sigma}} dz \right]^{\sigma} - 1$$

$$\frac{\partial L}{\partial c_t(z)} = p_t(z) - P_{Ht} \frac{1}{\sigma} \left[\left(\frac{1}{a} \right)^{\frac{1}{\sigma}} \int_0^a c_t(z)^{\frac{1}{\sigma}} dz \right]^{\sigma-1} \left(\frac{1}{a} \right)^{\frac{1}{\sigma}} c_t(z)^{\frac{1}{\sigma}-1} = 0$$

$$p_t(z) = P_{Ht} \left[\left(\frac{1}{a} \right)^{\frac{1}{\sigma}} \int_0^a c_t(z)^{\frac{1}{\sigma}} dz \right]^{\sigma-1} \left(\frac{1}{a} \right)^{\frac{1}{\sigma}} c_t(z)^{\frac{1}{\sigma}-1}$$

$$c_t(z)^{\frac{1}{\sigma}-1} = \frac{p_t(z)}{P_{Ht}} a^{\frac{1}{\sigma}}$$

$$c_t(z) = \frac{1}{a} \left(\frac{p_t(z)}{P_{Ht}} \right)^{\frac{\sigma}{\sigma-1}}$$

Substitute this expression into $C_{Ht} = 1$:

$$\left[\left(\frac{1}{a} \right)^{\frac{1}{\sigma}} \int_0^a \left(\frac{p_t(z)}{P_{Ht}} \right)^{\frac{\sigma}{\sigma-1}} \left(\frac{1}{a} \right)^{\frac{1}{\sigma}} dz \right]^{\sigma} = 1$$

$$\left[\left(\frac{1}{a} \right)^{\frac{1}{\sigma}} \left(\frac{1}{a} \right)^{\frac{1}{\sigma}} \int_0^a \left(\frac{p_t(z)}{P_{Ht}} \right)^{\frac{\sigma}{\sigma-1}} dz \right]^{\sigma} = 1$$

$$P_{Ht} \left[\int_0^a \left(\frac{1}{p_t(z)} \right)^{\frac{1}{\sigma-1}} dz \right]^{\sigma-1} = 1$$

$$\left[\int_0^a p_t(z)^{\frac{1}{\sigma-1}} dz \right]^{\sigma-1} = P_{Ht}^{\sigma}$$

$$\left[\int_0^a p_t(z)^{\frac{1}{\sigma-1}} dz \right]^{\frac{\sigma-1}{\sigma}} = P_{Ht}$$

$\left[\int_0^a p_t(z)^{\frac{1}{\sigma-1}} dz \right]^{\frac{\sigma-1}{\sigma}} = P_{Ht}$, which is the price index of goods produced by home firms (denoted by z) in the home country.

We can then write the demand for home firm z output by the representative household in the home country based on the above as:

$$c_t(z) = \frac{1}{a} \left(\frac{p_t(z)}{P_{Ht}} \right)^{\frac{\sigma}{\sigma-1}} \quad C_{Ht} = \frac{1}{a} \left(\frac{p_t(z)}{P_{Ht}} \right)^{\frac{\sigma}{\sigma-1}} \left(\frac{P_{Ht}}{p_t} \right)^{\frac{1}{\sigma-1}} a C_t = \left(\frac{p_t(z)}{P_{Ht}} \right)^{\frac{\sigma}{\sigma-1}} \left(\frac{P_{Ht}}{p_t} \right)^{\frac{1}{\sigma-1}} C_t^3.$$

Since there are a home households, the demand for home firm z output by all households in the home country is: $\left(\frac{p_t(z)}{P_{Ht}} \right)^{\frac{\sigma}{\sigma-1}} \left(\frac{P_{Ht}}{p_t} \right)^{\frac{1}{\sigma-1}} a C_t$.

²Note that this expression should be completely written as $c_t(z) = \frac{1}{a} \left(\frac{p_t(z)}{P_{Ht}} \right)^{\frac{\sigma}{\sigma-1}} C_{Ht}$ but we drop C_{Ht} because we imposed $C_{Ht} = 1$.

³Note that in this expression we should write $(C_t + G_t)$ to reflect the total demand made by the home country that comes from home consumers as well as home government. However, for the purpose of this derivation, we can omit G_t .

The demand for home z output by all households and government in the home country is: $(\frac{P_t(z)}{P_{Ht}})^{-1} (\frac{P_{Ht}}{P_t})^{-1} (aC_t + aG_t)$ assuming that the government spends G_t per capita. Notice: $a(C_t + G_t)$ is Y_t^d , i.e., demand for consumption basket in the home country. Note that in contrast to Ghironi, Lee, and Rebucci (2015) (GLR), we do not have Y_t^W . Note: The total per capita demand for consumption basket in the home country is $y_t^d = C_t + G_t$

The price index of goods produced by foreign firms in the home country can be derived by following the same steps. In this derivation, the home consumer's consumption basket, C_{Ft} , consists of goods produced by the foreign firms where we integrate from a to $1-a$ because there are $1-a$ foreign firms:

$$[\frac{1}{1-a} \int_a^1 P_t(z)^{-1} dz]^{-1} = P_{Ft} \text{ using consumption of goods produced by foreign firms in the home country, } C_{Ft} = [(\frac{1}{1-a}) \int_a^1 C_t(z)^{-1} dz]^{-1}$$

The derivation of the price index of goods produced in the foreign country (consisting of a price index of goods produced by home firms in the foreign country P_{Ht} , and a price index of goods produced by foreign firms in the foreign country P_{Ft}), P_t , yields:

$$P_t = [aP_{Ht}^{-1} + (1-a)P_{Ft}^{-1}]^{-1}$$

Note that the expressions for P_{Ht} , P_{Ft} , P_{Ht} and P_{Ft} (and, hence, P_t and P_t) are identical to GLR. However, since purchasing power parity does not hold in our model, we have to take the real exchange rate Q_t , into account.

$$Q_t = \frac{P_t}{P_t^*} \text{ where } P_t^* \text{ is the nominal exchange rate, and } P_t^* = [a(P_{Ht}^*)^{-1} + (1-a)(P_{Ft}^*)^{-1}]^{-1}.$$

$$\text{Then: } Q_t = \left[\frac{a(P_{Ht}^*)^{-1} + (1-a)(P_{Ft}^*)^{-1}}{aP_{Ht}^{-1} + (1-a)P_{Ft}^{-1}} \right]^{-1}$$

1.2 Household optimization

Start with the home household budget constraint in nominal terms in home currency:

$$(V_t + D_t + P_t D_t)X_t + (P_t^* V_t + D_t + P_t^* D_t)X_t^* + W_t L_t = V_t X_{t+1} + P_t^* V_t^* X_{t+1}^* + P_t C_t + P_t G_t,$$

where X_t denotes shares of the home firm, X_t^* denotes shares of the foreign firm, V_t is the price of the home firm's shares, V_t^* is the price of the foreign firm's shares, D_t is the dividend

With respect to x_{t+1} :

$$\frac{\partial}{\partial x_{t+1}} = -\lambda_t (v_t) + E_t f_{t+1} (v_{t+1} + d_{t+1} + d_{t+1})g = 0$$

$$C_{t+1}^{-1} v_t = E_t f_{t+1} C_{t+1}^{-1} (v_{t+1} + d_{t+1} + d_{t+1})g$$

home firm's dividends coming from

With respect to x_{t+1} :

$$\frac{\partial}{\partial x_{t+1}} = -\lambda_t (v_t) + E_t f_{t+1} (v_{t+1} + d_{t+1} + d_{t+1})g = 0$$

subject to:

$Y_t^s(z) = Y_t^d(z)$, which says that output supplied by the home firm in the home country has to equal this firm's output demanded in the home country,

and

$Y_t^s(z) = Y_t^d(z)$, which says that output supplied by the home firm in the foreign country has to equal this firm's output demanded in the foreign country.

To derive the optimal demand for labor by home firm, z , in the home country, we use

$Y_t^s(z) = Y_t^d(z)$. $Y_t^s(z)$ comes from the production function, i.e., $Y_t^s(z) = Z_t L_t(z)$. $Y_t^d(z)$

comes from the demand for home firm's good that was derived in Section 1.1. $Y_t^d(z) =$

$\left(\frac{P_t(z)}{P_Ht}\right) \left(\frac{P_Ht}{P_t}\right)^{\frac{1}{\sigma}} (aC_t + aG_t)$ (which is $Y_t^d(z) = \left(\frac{P_t(z)}{P_Ht}\right) \left(\frac{P_Ht}{P_t}\right)^{\frac{1}{\sigma}} Y_t^d$ because $aC_t + aG_t$ rep-

resents the demand by all home firms) TJ/F18 11.69 -1.339 Td [(H)-75(t)]TJ ET q 1 0 0 1 359.842 50

subject to:

$Y_t^s($

$p_t(z) = \frac{W_t}{Z_t}$, which is the price charged by the home firm in the foreign country.

For the foreign firm, z , the problem becomes:

$$\text{Max } p_t(z) Z_t^1 - Z_t \left(\frac{p_t(z)}{P_{Ft}} \right) \left(\frac{P_{Ft}}{P_t} \right)^{\frac{1}{\sigma}} \frac{a(C_t + G_t)}{Z_t} + \lambda_t p_t(z) Z_t \left(\frac{p_t(z)}{P_{Ft}} \right) \left(\frac{P_{Ft}}{P_t} \right)^{\frac{1}{\sigma}} \frac{(1-a)(C_t + G_t)}{Z_t} \\ W_t \left(\frac{p_t(z)}{P_{Ft}} \right) \left(\frac{P_{Ft}}{P_t} \right)^{\frac{1}{\sigma}} \frac{a(C_t + G_t)}{Z_t} - \lambda_t W_t \left(\frac{p_t(z)}{P_{Ft}} \right) \left(\frac{P_{Ft}}{P_t} \right)^{\frac{1}{\sigma}} \frac{(1-a)(C_t + G_t)}{Z_t}$$

Take the derivative with respect to $p_t(z)$:

$$(1) = \frac{W_t}{Z_t p_t(z)}$$

$p_t(z) = \frac{W_t}{Z_t}$, which is the price charged by the foreign firm in the home country.

Take the derivative with respect to $p_t(z)$:

$$(1) = \frac{W_t}{Z_t p_t(z)}$$

$p_t(z) = \frac{W_t}{Z_t}$, which is the price charged by the foreign firm in the foreign country.

In equilibrium, $p_t(z) = P_{Ht}$, which says that price charged by home firms in home country equals the price index for goods produced by home firms. Similarly, $p_t(z) = P_{Ht}$ for price charged by home firms in the foreign country, $p_t(z) = P_{Ft}$ for price charged by foreign firms in the home country, and $p_t(z) = P_{Ft}$ for price charged by foreign firms in the foreign country.

Therefore:

$P_{Ht} = \frac{W_t}{Z_t}$ for price index of goods produced by home firms in the home country,

$P_{Ht} = \frac{W_t}{Z_t}$ for price index of goods produced by home firms in the foreign country,

$P_{Ft} = \frac{W_t}{Z_t}$ for price index of goods produced by foreign firms in the home country,

and

$P_{Ft} = \frac{W_t}{Z_t}$ for price index of goods produced by foreign firms in the foreign country.

Then, we can write expressions for relative prices:

$RP_t = \frac{p_t(z)}{P_t} = \frac{P_{Ht}}{P_t} = \frac{W_t}{Z_t}$ for price charged by a home firm in the home country relative to the home country's price level in units of the home country consumption,

$RP_t = \frac{p_t(z)}{P_t} = \frac{P_{Ht}}{P_t} = \frac{W_t}{Z_t}$ for price charged by a home firm in the foreign country

relative to the foreign country's price of the foreign good. In other words, the relative price of the foreign good in the home country is $\frac{p_t(z)}{P_t}$. In the foreign country, the relative price of the foreign good is $\frac{P_{Ft}}{P_t}$. The relative price of the foreign good in the home country is $\frac{p_t(z)}{P_t} = \frac{P_{Ft}}{P_t} = \frac{w_t}{Z_t}$.

relative to the home country's price of the foreign good. In other words, the relative price of the foreign good in the home country is $\frac{p_t(z)}{P_t}$. In the foreign country, the relative price of the foreign good is $\frac{P_{Ft}}{P_t}$. The relative price of the foreign good in the home country is $\frac{p_t(z)}{P_t} = \frac{P_{Ft}}{P_t} = \frac{w_t}{Z_t}$.

Note that the small case letter, w_t , denotes real wage as opposed to the large case letter W that denotes nominal wage.

The optimal labor demands can be derived from the first order conditions with relative prices as follows. The optimal demand for labor by a home country firm is given by

$$L_t(z) = RP_t^{-1} \frac{a(C_t + G_t)}{Z_t} z = RP_t^{-1} \frac{a(C_t + G_t)}{Z_t} z$$

$$(1 - a)L_t(z) = (1 - a)RP_t \frac{a(C_t + G_t)}{Z_t^{1-a}}$$

Per capita labor demand by all foreign firms in home country is:

$$\frac{1-a}{a}L_t(z) = \frac{1-a}{a}RP_t \frac{a(C_t + G_t)}{Z_t^{1-a}}$$

where we again divide by a because there are a households in the home country.

There are $(1 - a)$ home firms in the foreign country, so the optimal demand for labor by all home firms in the foreign country is:

$$aL_t(z) = aRP_t \frac{(1 - a)(C_t + G_t)}{Z_t^{1-a}}$$

Per capita labor demand by all home firms in foreign country is:

$$\frac{a}{1-a}L_t(z) = \frac{a}{1-a}RP_t \frac{(1 - a)(C_t + G_t)}{Z_t^{1-a}}$$

where we divide by $(1 - a)$ because there are $(1 - a)$ households in the home country.

There are $(1 - a)$ foreign firms in the foreign country, so the optimal demand for labor by all foreign firms in the foreign country is:

$$(1 - a)L_t(z) = (1 - a)RP_t \frac{(1 - a)(C_t + G_t)}{Z_t^{1-a}}$$

Total per capita labor demand by all foreign firms in the foreign country is:

$$\frac{1-a}{1-a}L_t(z) = \frac{1-a}{1-a}RP_t \frac{(1 - a)(C_t + G_t)}{Z_t^{1-a}}$$

where we again divide by $(1 - a)$ because there are $(1 - a)$ households in the home country.

1.4 Net foreign assets (NFA) law of motion

Start with the home household budget constraint in units of the home country's consumption basket from Section 1.2:

$$(v_t + d_t + d_t)x_t + (v_t + d_t + d_t)x_t + w_t L_t = v_t x_{t+1} + v_t x_{t+1} + C_t + G_t$$

Then:

$$(v_t + d_t + d_t^*)x_t + (v_t + d_t + d_t^*)x_t + w_t L_t = v_t x_{t+1} + nfa_{t+1} + \frac{1-a}{a} v_t x_{t+1} + C_t + G_t$$

where net foreign assets, nfa_{t+1} , is defined as $nfa_{t+1} = v_t x_{t+1} - \frac{1-a}{a} v_t x_{t+1}$, i.e., the value of home holdings of foreign shares minus the value of foreign holdings of home shares adjusted for population sizes of home and foreign countries, i.e., a and $1-a$, respectively, as in GLR.

We define return on holding home equity as $R_t = \frac{v_t + d_t + d_t^*}{v_{t-1}}$ and return on holding foreign equity as $R_t^* = \frac{v_t + d_t + d_t^*}{v_{t-1}}$ in Section 1.2, so:

$$v_t x_{t+1} + nfa_{t+1} + \frac{1-a}{a} v_t x_{t+1} + C_t + G_t = \frac{(v_t + d_t + d_t^*) v_{t-1}}{v_{t-1}} x_t + \frac{(v_t + d_t + d_t^*) v_{t-1}}{v_{t-1}} x_t + w_t L_t$$

$$v_t x_{t+1} + nfa_{t+1} + \frac{1-a}{a} v_t x_{t+1} + C_t + G_t = R_t v_{t-1} x_t + R_t^* v_{t-1} x_t + w_t L_t$$

$$nfa_{t+1} = v_t x_{t+1} - \frac{1-a}{a} v_t x_{t+1} + R_t v_{t-1} x_t + R_t^* v_{t-1} x_t + w_t L_t - C_t - G_t$$

$$nfa_{t+1} = v_t (x_{t+1} + \frac{1-a}{a} x_{t+1}) + R_t v_{t-1} x_t + R_t^* v_{t-1} x_t + w_t L_t - C_t - G_t$$

$$nfa_{t+1} = v_t + R_t v_{t-1} x_t + R_t^* v_{t-1} x_t + w_t L_t - C_t - G_t$$

where market clearing condition $a x_{t+1} + (1-a) x_{t+1} = a$ was used to obtain $x_{t+1} = 1 - \frac{1-a}{a} x_{t+1}$ as in GLR.

$$nfa_{t+1} = v_t + R_t v_{t-1} x_t + R_t^* v_{t-1} (1 - \frac{1-a}{a} x_t) + w_t L_t - C_t - G_t \text{ where we used } x_t = 1 - x_t \frac{1-a}{a}.$$

$$nfa_{t+1} = v_t + R_t v_{t-1} x_t + R_t^* v_{t-1} - R_t^* v_{t-1} \frac{1-a}{a} x_t + w_t L_t - C_t - G_t$$

$$nfa_{t+1} = v_t + R_t v_{t-1} x_t + v_t + d_t + d_t^* - R_t^* v_{t-1} \frac{1-a}{a} x_t + w_t L_t - C_t - G_t$$

$$nfa_{t+1} = R_t v_{t-1} x_t - R_t^* v_{t-1} \frac{1-a}{a} x_t + y_t - C_t - G_t$$

where $y_t = d_t + d_t^* + w_t L_t$, which differs from GLR due to the additional term d_t^* . Note that we assume that the dividend of the home firm producing in the foreign country, d_t^* , is a part of the home country GDP, i.e., we assume that firms repatriate profits to their countries of origin for distribution to domestic and foreign shareholders.

$$nfa_{t+1} = R_t v_{t-1} x_t - R_t^* v_{t-1} x_t + R_t v_{t-1} x_t - R_t^* v_{t-1} \frac{1-a}{a} x_t + y_t - C_t - G_t$$

Define excess return from holding foreign equity $R_t^D = R_t - R_t^*$ and portfolio holding

$$x_t = v_{t-1} x_t:$$

$$nfa_{t+1} = R_t^D x_t + R_t v_{t-1} x_t - R_t^* v_{t-1} \frac{1-a}{a} x_t + y_t - C_t - G_t$$

$$nfa_{t+1} = R_t^D x_t + R_t nfa_t + y_t - C_t - G_t$$

where definition $nfa_t = v_{t-1} x_t - \frac{1-a}{a} v_{t-1} x_t$ was used.

This is identical to GLR except the definitions of R_t and R_t^* , and hence R_t^D , differ as explained above. This is in units of home consumption.

Similar derivations can be done to obtain the NFA law of motion for the foreign household:

$$nfa_{t+1}^f = R_t^{Df} nfa_t^f + R_t^f nfa_t^f + y_t^f - C_t^f$$

Derivation of home GDP, y_t , i.e., output produced by home and foreign firms in the home country:

$$y_t = RP_t Z_t L_t + RP_t Z_t^{-1} Z_t L_t = \frac{w_t}{1} Z_t L_t + \frac{w_t}{1 Z_t^{-1} Z_t} Z_t L_t = \frac{w_t}{1} (L_t + L_t),$$

which is in units of home country consumption.

Derivation of foreign GDP, y_t^* , i.e., output produced by home and foreign firms in the foreign country:

$$y_t^* = RP_t^* Z_t^* L_t^* + RP_t^* Z_t^* L_t^* = \frac{w_t^*}{1 Z_t^* Z_t^{*1}} Z_t^* L_t^* + \frac{w_t^*}{1 Z_t^*} Z_t^* L_t^* = \frac{w_t^*}{1} (L_t^* + L_t^*),$$

which is in units of foreign country consumption.

Expression for $\frac{y_t^*}{y_t}$:

$$\frac{y_t^*}{y_t} = \frac{RP_t^* Z_t^* L_t^* + RP_t^* Z_t^{*1} Z_t^* L_t^*}{RP_t Z_t L_t + RP_t Z_t^{-1} Z_t L_t} = \frac{\frac{w_t^*}{1} (L_t^* + L_t^*)}{\frac{w_t}{1} (L_t + L_t)} = \frac{w_t^* (L_t^* + L_t^*)}{w_t (L_t + L_t)}$$

Note that we should use the real exchange rate in the relative GDP expression but it cancels because it appears on both sides of the equation $\frac{y_t^*}{Q_t y_t} = \frac{w_t^* (L_t^* + L_t^*)}{Q_t w_t (L_t + L_t)}$

Next, expressions for w_t , w_t^* , $(L_t + L_t)$ and $(L_t^* + L_t^*)$ are obtained. To get w_t , home labor supply and home labor demand are equated. Home labor supply was derived in Section 1.2 from home household FOCs as $L_t^s = \left(\frac{C_t}{w_t} \right)^\alpha$. Home labor demand was derived above from firm FOCs in Section 1.3 as $L_t^d = RP_t^{-1} \frac{a(C_t + G_t)}{w_t}$

(c¹)

$$\begin{aligned}
\frac{y_t}{y_t} &= \left[\frac{C_t + G_t}{C_t + G_t} \right]^{\frac{1}{1+\alpha}} \left[\frac{aZ_t^{1-\alpha} + (1-a)(Z_t^1 - Z_t)^{1-\alpha}}{a(Z_t - Z_t^1)^{1-\alpha} + (1-a)Z_t^{1-\alpha}} \right]^{\frac{1+\alpha}{1-\alpha}} \left[\frac{C_t + G_t}{C_t + G_t} \right]^{\frac{1+\alpha}{1-\alpha}} \left[\frac{aZ_t^{1-\alpha} + (1-a)(Z_t^1 - Z_t)^{1-\alpha}}{a(Z_t - Z_t^1)^{1-\alpha} + (1-a)Z_t^{1-\alpha}} \right]^{\frac{1+\alpha}{1-\alpha}} = \\
&= \left[\frac{C_t + G_t}{C_t + G_t} \right]^{\frac{(1+\alpha)(1+\alpha)}{1-\alpha}} + \left[\frac{aZ_t^{1-\alpha} + (1-a)(Z_t^1 - Z_t)^{1-\alpha}}{a(Z_t - Z_t^1)^{1-\alpha} + (1-a)Z_t^{1-\alpha}} \right]^{\frac{(1+\alpha)(1-\alpha)(1+\alpha)(1-\alpha)}{(1-\alpha)(1+\alpha)}} = \\
&= \frac{C_t + G_t}{C_t + G_t}
\end{aligned}$$

1.6 More on real exchange rate, Q_t

From Section 1.1: $Q_t = \left[\frac{a({}^t P_{Ht})^{1-\alpha} + (1-a)({}^t P_{Ft})^{1-\alpha}}{aP_{Ht}^{1-\alpha} + (1-a)P_{Ft}^{1-\alpha}} \right]^{\frac{1}{1-\alpha}}$.

$$Q_t^{1-\alpha} = \frac{a({}^t P_{Ht})^{1-\alpha} + (1-a)({}^t P_{Ft})^{1-\alpha}}{aP_{Ht}^{1-\alpha} + (1-a)P_{Ft}^{1-\alpha}}$$

Use expressions for price indices:

$$\begin{aligned}
Q_t^{1-\alpha} &= \frac{a\left({}^t \frac{W_t}{Z_t} \frac{W_t}{Z_t}\right)^{1-\alpha} + (1-a)\left({}^t \frac{W_t}{Z_t}\right)^{1-\alpha}}{a\left(\frac{W_t}{Z_t}\right)^{1-\alpha} + (1-a)\left(\frac{W_t}{Z_t}\right)^{1-\alpha}} = \left(\frac{{}^t W_t}{W_t}\right)^{1-\alpha} \frac{a(Z_t - Z_t^1)^{1-\alpha} + (1-a)Z_t^{1-\alpha}}{aZ_t^{1-\alpha} + (1-a)(Z_t^1 - Z_t)^{1-\alpha}} = \\
&= \left(\frac{{}^t W_t P_t}{P_t}\right)^{1-\alpha} \frac{a(Z_t - Z_t^1)^{1-\alpha} + (1-a)Z_t^{1-\alpha}}{aZ_t^{1-\alpha} + (1-a)(Z_t^1 - Z_t)^{1-\alpha}}
\end{aligned}$$

$$\begin{aligned}
&)^{1-\alpha} \frac{a(Z_t - Z_t^1)^{1-\alpha} + (1-a)Z_t^{1-\alpha}}{aZ_t^{1-\alpha} + (1-a)(Z_t^1 - Z_t)^{1-\alpha}}
\end{aligned}$$

$$\left(\frac{a}{1-a}\right)^{\frac{!+!}{!-1} \frac{(!-1)}{!}} [Z$$

Similarly, foreign GDP y_t^* , i.e., output produced by home and foreign firms in the foreign country, equals $y_t^* = \frac{1}{\alpha} (w_t L_t + w_t^* L_t^*)$ in units of foreign country consumption. Labor income, therefore, equals $w_t L_t$. In units of home country consumption, this is $\frac{1}{\alpha} y_t^* Q_t$. The profit of foreign firms, i.e., the profit generated by foreign firms in home and foreign countries, $d_t + d_t^*$, in units of home country consumption is then $\frac{1}{\alpha} y_t^* Q_t$, which again shows that the share of firm profits, i.e., the dividend income, in the foreign GDP is a constant proportion $\frac{1}{\alpha}$.

$$E_t(\mathcal{C}_{t+1}^D, \mathcal{C}_t^D) = E_t(\mathcal{C}_{t+1}, \mathcal{C}_t)$$

2.2 Log-linearize expression from Section 1.6 and find elasticities of \mathcal{C}_t^D

This derivation finds elasticities of \mathcal{C}_t^D :

$$\frac{C_t + G_t}{C_t + G_t} \left(\frac{C_t}{C_t} \right)' = \left[\frac{aZ_t^{1-\alpha} + (1-\alpha)(Z_t^1 - Z_t)^{1-\alpha}}{a(Z_t Z_t^{-1})^{1-\alpha} + (1-\alpha)Z_t^{1-\alpha}} \right]^{1+\frac{1}{\sigma}}$$

$$\log(C_t + G_t) - \log(C_t + G_t) + \frac{1+\frac{1}{\sigma}}{1-\alpha} [\log(aZ_t^{1-\alpha} + (1-\alpha)(Z_t^1 - Z_t)^{1-\alpha}) - \log(aZ_t Z_t^{-1})^{1-\alpha} + (1-\alpha)Z_t^{1-\alpha}]$$

$$\frac{dC_t + dG_t}{C_t + G_t} - \frac{dC_t + dG_t}{C_t + G_t} + \frac{1+\frac{1}{\sigma}}{1-\alpha} \left[\frac{dC_t}{C_t} - \frac{dC_t}{C_t} \right] = \frac{1+\frac{1}{\sigma}}{1-\alpha} [a^{1-\alpha} dZ_t + (1-\alpha)^{1-\alpha} ((1-\alpha) dZ_t + dZ_t) - [a^{1-\alpha} (dZ_t + (1-\alpha) dZ_t) + (1-\alpha)^{1-\alpha} dZ_t]]$$

Use $Z = Z$, which is true in the symmetric steady state. Normalize $Z = Z$ to 1.

$$\frac{dC_t \frac{C}{C+G} + dG_t \frac{G}{C+G}}{\frac{C}{C+G} + \frac{G}{C+G}} - \frac{dC_t \frac{C}{C+G} + dG_t \frac{G}{C+G}}{\frac{C}{C+G} + \frac{G}{C+G}} + \frac{1+\frac{1}{\sigma}}{1-\alpha} \mathcal{C}_t^D = \frac{1+\frac{1}{\sigma}}{1-\alpha} [a^{1-\alpha} (1-\alpha) \mathcal{Z}_t + (1-\alpha)^{1-\alpha} ((1-\alpha) \mathcal{Z}_t + \mathcal{Z}_t) - [a^{1-\alpha} (1-\alpha) (\mathcal{Z}_t + (1-\alpha) \mathcal{Z}_t) + (1-\alpha)^{1-\alpha} \mathcal{Z}_t]]$$

$$\frac{C}{C+G} (\mathcal{C}_t - \mathcal{C}_t) + \frac{G}{C+G} (\mathcal{C}_t - \mathcal{C}_t) + \frac{1+\frac{1}{\sigma}}{1-\alpha} \mathcal{C}_t^D = \frac{1+\frac{1}{\sigma}}{1-\alpha} [a^{1-\alpha} (1-\alpha) \mathcal{Z}_t + (1-\alpha)^{1-\alpha} ((1-\alpha) \mathcal{Z}_t + \mathcal{Z}_t) - [a^{1-\alpha} (1-\alpha) (\mathcal{Z}_t + (1-\alpha) \mathcal{Z}_t) + (1-\alpha)^{1-\alpha} \mathcal{Z}_t]]$$

Use $y = C + G$. Since $y = 1$, $C + G = 1$ and $C = 1 - G$. Then,

$$(1-G)(\mathcal{C}_t - \mathcal{C}_t) + G(\mathcal{C}_t - \mathcal{C}_t) + \frac{1+\frac{1}{\sigma}}{1-\alpha} \mathcal{C}_t^D = \frac{1+\frac{1}{\sigma}}{1-\alpha} [a^{1-\alpha} (1-\alpha) (1-\mathcal{Z}_t) + (1-\alpha)^{1-\alpha} ((1-\alpha) \mathcal{Z}_t - (1-\alpha) \mathcal{Z}_t) - [a^{1-\alpha} (1-\alpha) (1-\mathcal{Z}_t) + (1-\alpha)^{1-\alpha} (1-\mathcal{Z}_t)]]$$

$$(1-G)\mathcal{C}_t^D + G\mathcal{C}_t^D + \frac{1+\frac{1}{\sigma}}{1-\alpha} \mathcal{C}_t^D = \frac{1+\frac{1}{\sigma}}{1-\alpha} (1-\alpha) (1-\mathcal{Z}_t) (\mathcal{Z}_t - \mathcal{Z}_t)$$

$$(1-G)\mathcal{C}_t^D + G\mathcal{C}_t^D + \frac{1+\frac{1}{\sigma}}{1-\alpha} \mathcal{C}_t^D = (1+\frac{1}{\sigma}) (1-\mathcal{Z}_t) \mathcal{Z}_t^D$$

$$(1-G + \frac{1+\frac{1}{\sigma}}{1-\alpha}) \mathcal{C}_t^D = (1+\frac{1}{\sigma}) (1-\mathcal{Z}_t) \mathcal{Z}_t^D - G\mathcal{C}_t^D$$

$$\mathcal{C}_t^D = \frac{(1+\frac{1}{\sigma})(1-\mathcal{Z}_t)}{1-G + \frac{1+\frac{1}{\sigma}}{1-\alpha}} \mathcal{Z}_t^D - \frac{G}{1-G + \frac{1+\frac{1}{\sigma}}{1-\alpha}} \mathcal{C}_t^D$$

$$\mathcal{C}_t^D = \frac{1}{C^D Z^D} \mathcal{Z}_t^D + \frac{1}{C^D G^D} \mathcal{C}_t^D$$

If $G = 0$ (i.e., no fiscal shocks), $\mathcal{C}_t^D = \frac{(1+\frac{1}{\sigma})(1-\mathcal{Z}_t)}{1+\frac{1}{\sigma}} \mathcal{Z}_t^D$

If $G = 0$ and $\sigma = 0$ (i.e., inelastic labor) $\mathcal{C}_t^D = (1-\mathcal{Z}_t) \mathcal{Z}_t^D$

If $G = 0$, $\sigma = 0$, and $\alpha = 1$, $\mathcal{C}_t^D = 0$.

If $G = 0$, $\sigma = 0$, and $\alpha = 0$, $\mathcal{C}_t^D = \mathcal{Z}_t^D$.

If $G = 0$ and $\alpha = 1$, $\mathcal{C}_t^D = 0$ regardless of σ .

If $G \neq 0$ and $\sigma = 0$, $\mathcal{C}_t^D = \frac{(1-\mathcal{Z}_t)}{1-G} \mathcal{Z}_t^D - \frac{G}{1-G} \mathcal{C}_t^D$

If $G \neq 0$, $\sigma = 0$ and $\alpha = 1$, $\mathcal{C}_t^D = \frac{G}{1-G} \mathcal{C}_t^D$. If $\alpha = 0$, $\mathcal{C}_t^D = \frac{1}{1-G} \mathcal{Z}_t^D - \frac{G}{1-G} \mathcal{C}_t^D$.

2.3 Find elasticities of Q_t

This derivation uses the log-linearized Euler equations from Section 2.1 and c_t^D from Section 2.2 to find elasticities of Q_t :

$$E_t(c_{t+1}^D - c_t^D) = E_t(Q_{t+1} - Q_t) \text{ from Section 2.1.}$$

Combine with $c_t^D = c^D Z_t^D + c^D G^D c_t^D$ from 2.2.

$$E_t(Q_{t+1} - Q_t) = E_t[c^D Z_{t+1}^D - c^D Z_t^D] + c^D G^D c_t^D$$

Log-linearized: $b_t^{\text{total};D} = b_t^D - w_t^D$.

2.6 Log-linearize NFA LOM

This derivation uses the NFA LOM from Section 1.4 to find the solution for nfa_{t+1} :

$$nfa_{t+1} = R_t^D b_t + R_t nfa_t + (1 - a)[(y_t - Q_t y_t^f) - (C_t - Q_t C_t^f) - (G_t - Q_t G_t^f)]$$

$$dnfa_{t+1} = dR_t^D b_t + R_t^D db_t + dR_t nfa_t + R_t dnfa_t + (1 - a)[dy_t - (dQ_t y_t^f + Q_t dy_t^f) - (dC_t - (dQ_t C_t^f + Q_t dC_t^f)) - (dG_t - (dQ_t G_t^f + Q_t dG_t^f))]$$

Use $R^D = 0$ and $nfa = 0$:

$$dnfa_{t+1} = dR_t^D b_t + R_t dnfa_t + (1 - a)[dy_t - (dQ_t y_t^f + Q_t dy_t^f) - (dC_t - (dQ_t C_t^f + Q_t dC_t^f)) - (dG_t - (dQ_t G_t^f + Q_t dG_t^f))]$$

Use $Q = 1$ because it holds in the symmetric steady state, and net foreign assets equal 0:

$$dnfa_{t+1} = dR_t^D b_t + R_t dnfa_t + (1 - a)[(dy_t - (dQ_t y_t^f + dy_t^f) - (dC_t - (dQ_t C_t^f + dC_t^f)) - (dG_t - (dQ_t G_t^f + dG_t^f))]$$

Notice that we are subtracting dy_t and dy_t^f that are in units of home consumption and foreign consumption, respectively. We can subtract these terms because we already accounted for the different units by including the real exchange rate. This works because in the symmetric steady state, the real exchange rate terms drop out ($Q = 1$). This is used later on in other derivations, for example, the derivation of the differential in equity values b_t^D .

$$dnfa_{t+1} = dR_t^D b_t + R_t dnfa_t + (1 - a)[(dy_t^D - dQ_t y_t^f) - (dC_t^D - dQ_t C_t^f) - (dG_t^D - dQ_t G_t^f)]$$

Divide by C . Use $C = 1 - G$, which comes from $y = C + G$ combined with $y = 1$:

$$\frac{dnfa_{t+1}}{C} = \frac{dR_t^D}{1 - G} b_t + \frac{R_t dnfa_t}{C} + (1 - a)[\left(\frac{dy_t^D}{1 - G} - \frac{dQ_t y_t^f}{1 - G}\right) - \left(\frac{dC_t^D}{C} - \frac{dQ_t C_t^f}{C}\right) - \left(\frac{dG_t^D}{1 - G} - \frac{dQ_t G_t^f}{1 - G}\right)]$$

$$nfa_{t+1} = \frac{dR_t^D}{1 - G} R b_t$$

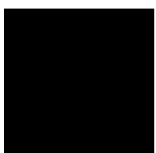
$$nfa_{t+1} = \frac{1}{(1-G)} R_t^D + \frac{1}{1-G} nfa_t + \frac{1}{1-G} a^D (1-a) c_t^D \left[\frac{(1-a)G}{1-G} c_t^D + (1-a) \left[\frac{q_t}{1-G} + \frac{q_t(1-G^f)}{1-G} + \frac{q_t G^f}{1-G} \right] \right]$$

$$nfa_{t+1} = \frac{1}{(1-G)} R_t^D$$

$$\frac{dv_t}{v} = \frac{dE_t v_{t+1}}{v} + \frac{dE_t d_{t+1}}{v} + \frac{dE_t d_{t+1}}{v}$$

$$v_t = E_t v_{t+1} + E_t \frac{d_{t+1}}{v} + E_t \frac{d_{t+1}}{v}$$

From Section 1.7, the following holds: $d_t + d_t = 1y_t$. Due to the assumption $y_t = 1$, it is possible to write: $d_t + d_t = 1$. In steady state, the Euler equation for home shares becomes $v = v + d + d$, which becomes $v = v + 1$ which can be written as $v(1 -) = -$



Next, we obtain an expression for \bar{d}_{t+1}^D . Here, we take advantage of the useful properties from Section 1.7. Since $\bar{d}_t = d_t + d_t = \frac{1}{y_t} Q_t$ and $\bar{d}_t = d_t + d_t = \frac{1}{y_t} Q_t$ in units of home country consumption, it is possible to write $\frac{\bar{d}_t}{d_t} = \frac{d_t + d_t}{d_t} = \frac{1}{y_t} \frac{Q_t}{Q_t}$, which means $\frac{\bar{d}_t}{d_t} = \frac{y_t}{y_t Q_t}$.

Roll it forward by one period: $\frac{\bar{d}_{t+1}}{d_{t+1}} = \frac{y_{t+1}}{y_{t+1} Q_{t+1}}$.

Log-linearizing gives $\bar{d}_{t+1}^D = \bar{d}_{t+1} + \bar{d}_{t+1}^D$.

Substitute into \bar{d}_t^D :

$$\bar{d}_t^D = E_t[\bar{d}_{t+1}^D + (1 - \rho)(\bar{d}_{t+1} - (\bar{d}_{t+1} + \bar{d}_{t+1}^D))]$$

$$\text{Notice: This combines } E_t \bar{d}_{t+1}^D = 0 \text{ and } \bar{d}_t^D = [\bar{d}_t + (1 - \rho)(\bar{d}_t - (\bar{d}_t + \bar{d}_t^D))] + \bar{d}_t^D = [\bar{d}_t + (1 - \rho)(\bar{d}_t - \bar{d}_t^D)] + \bar{d}_t^D$$

$$\bar{d}_t^D = E_t[\bar{d}_{t+1}^D + (1 - \rho)(\bar{d}_{t+1} - \bar{d}_{t+1}^D)]$$

$$\bar{d}_t^D = E_t[\bar{d}_{t+1}^D + (1 - \rho)(y^D Z^D \bar{z}_{t+1}^D + y^D G^D \bar{g}_{t+1}^D - \rho Z^D \bar{z}_{t+1}^D - \rho G^D \bar{g}_{t+1}^D)]$$

$$\bar{d}_t^D = E_t[\bar{d}_{t+1}^D + (1 - \rho)(y^D Z^D \bar{z}_{t+1}^D + y^D G^D \bar{g}_{t+1}^D - \frac{1}{\rho} C^D Z^D \bar{z}_{t+1}^D - \frac{1}{\rho} C^D G^D \bar{g}_{t+1}^D)]$$

$$\bar{d}_t^D = E_t[\bar{d}_{t+1}^D + (1 - \rho)((y^D Z^D - \frac{1}{\rho} C^D Z^D) \bar{z}_{t+1}^D + (y^D G^D - \frac{1}{\rho} C^D G^D) \bar{g}_{t+1}^D)]$$

$$\bar{d}_t^D = v^D Z^D \bar{z}_t^D + v^D G^D \bar{g}_t^D$$

$$v^D Z^D \bar{z}_t^D + v^D G^D \bar{g}_t^D = E_t[\bar{d}_{t+1}^D + (1 - \rho)((y^D Z^D - \frac{1}{\rho} C^D Z^D) \bar{z}_{t+1}^D + (y^D G^D - \frac{1}{\rho} C^D G^D) \bar{g}_{t+1}^D)]$$

$$v^D Z^D \bar{z}_t^D + v^D G^D \bar{g}_t^D = (v^D Z^D Z \bar{z}_t^D + v^D G^D G \bar{g}_t^D) + (1 - \rho)[(y^D Z^D - \frac{1}{\rho} C^D Z^D) Z \bar{z}_t^D + (y^D G^D - \frac{1}{\rho} C^D G^D) G \bar{g}_t^D]$$

$$\text{where we used } \bar{z}_{t+1}^D = Z \bar{z}_t^D + b_{Z^D t+1} \text{ and } \bar{g}_{t+1}^D = G \bar{g}_t^D + b_{G^D t+1}$$

Match the coefficients:

$$v^D Z^D = v^D Z^D Z + (1 - \rho)(y^D Z^D - \frac{1}{\rho} C^D Z^D) Z$$

$$(1 - \rho) Z v^D Z^D = (1 - \rho)(y^D Z^D - \frac{1}{\rho} C^D Z^D) Z$$

$$v^D Z^D = \frac{(1 - \rho) Z (y^D Z^D - \frac{1}{\rho} C^D Z^D)}{1 - Z}$$

$$v^D G^D = v^D G^D G + (1 - \rho)(y^D G^D - \frac{1}{\rho} C^D G^D) G$$

$$(1 - \rho) G v^D G^D = (1 - \rho)(y^D G^D - \frac{1}{\rho} C^D G^D) G$$

$$v^D G^D = \frac{(1 - \rho) G (y^D G^D - \frac{1}{\rho} C^D G^D)}{1 - G}$$

2.8 Show that excess return \bar{R}_t^D is a linear function of innovations to relative productivity and government spending

From Section 2.7:

$$\bar{R}_{t+1}^D = [\bar{v}_{t+1}^D + (1 - \delta)] \bar{v}_{t+1}^D + b$$

If $G \notin 0$, $' = 0$ and $= 1$: $R_{t+1}^D = \frac{(1-z)(1-G)}{(1-z)(1-G)} b_{t+1}^D = \frac{(1-G)}{(1-G)(1-G)} b_{t+1}^D$

2.9 2nd-order approximation of the portfolio part of the model

From household FOCs for :

$$C_t^{-1} = E_t f_{C_{t+1}}^{-1} R_{t+1} g, \text{ which can be written as: } \frac{C_t^{-1}}{C_{t+1}^{-1}} = E_t f_{C_{t+1}}^{-1} R_{t+1} g$$

$$C_t^{-1} = E_t f_{C_{t+1}}^{-1} R_{t+1} g, \text{ which can be written as: } \frac{C_t^{-1}}{C_{t+1}^{-1}} = E_t f_{C_{t+1}}^{-1} R_{t+1} g$$

Equating these two expressions gives $u E_t(C_{t+1}^{-1} R_{t+1}) = E_t(C_{t+1}^{-1} R_{t+1})$, which can be written as $E_t(C_{t+1}^{-1} R_{t+1}) - E_t(C_{t+1}^{-1} R_{t+1}) = 0$

Take second-order approximation and evaluate it at steady state:

$$E_t(C_{t+1}^{-1} dC_{t+1} R_{t+1}) + E_t(C_{t+1}^{-1} dR_{t+1}) - E_t(C_{t+1}^{-1} dC_{t+1} R_{t+1}) - E_t(C_{t+1}^{-1} dR_{t+1}) +$$

$$+ \frac{1}{2} [\frac{1}{2} (\frac{1}{2} - 1) C_{t+1}^{-1} d^2 C_{t+1} R_{t+1} + C_{t+1}^{-1} 0 + 2 (\frac{1}{2}) C_{t+1}^{-1} dC_{t+1} dR_{t+1}]$$

$$\frac{1}{2} [\frac{1}{2} (\frac{1}{2} - 1) C_{t+1}^{-1} d^2 C_{t+1} R_{t+1} - C_{t+1}^{-1} 0 - 2 (\frac{1}{2}) C_{t+1}^{-1} dC_{t+1} dR_{t+1}] =$$

$$= E_t(C_{t+1}^{-1} dC_{t+1} R_{t+1}) + E_t(C_{t+1}^{-1} dR_{t+1}) - E_t(C_{t+1}^{-1} dC_{t+1} R_{t+1}) - E_t(C_{t+1}^{-1} dR_{t+1}) +$$

$$+ \frac{1}{2} [\frac{1}{2} (\frac{1}{2} - 1) C_{t+1}^{-1} d^2 C_{t+1} R_{t+1} + 2 (\frac{1}{2}) C_{t+1}^{-1} dC_{t+1} dR_{t+1}]$$

$$\frac{1}{2} [\frac{1}{2} (\frac{1}{2} - 1) C_{t+1}^{-1} d^2 C_{t+1} R_{t+1} + 2 (\frac{1}{2}) C_{t+1}^{-1} dC_{t+1} dR_{t+1}] =$$

$$= E_t(C_{t+1}^{-1} dR_{t+1}) - E_t(C_{t+1}^{-1} dR_{t+1}) + \frac{1}{2} [2 (\frac{1}{2}) C_{t+1}^{-1} dC_{t+1} dR_{t+1}] - \frac{1}{2} [2 (\frac{1}{2}) C_{t+1}^{-1} dC_{t+1} dR_{t+1}]$$

$$= E_t(C_{t+1}^{-1} dR_{t+1}) - E_t(C_{t+1}^{-1} dR_{t+1}) + [\frac{1}{2} C_{t+1}^{-1} dC_{t+1} dR_{t+1}] - [\frac{1}{2} C_{t+1}^{-1} dC_{t+1} dR_{t+1}]$$

Divide by $C_{t+1}^{-1} dR_{t+1}$

Log-linearize: $R_{t+1} = Q_{t+1} Q_t + R_{t+1}^f$

Using this, simplify: $(\frac{1}{z} c_{t+1}^D R_{t+1}) - (\frac{1}{z} c_{t+1}^D R_{t+1}^f) - [(\frac{1}{z} c_{t+1}^D R_{t+1}^f) - (\frac{1}{z} c_{t+1}^D R_{t+1}^f)] = 0$

$$c_{t+1}^D R_{t+1} - c_{t+1}^D R_{t+1}^f - [c_{t+1}^D R_{t+1}^f - c_{t+1}^D R_{t+1}^f] = 0$$

$$c_{t+1}^D R_{t+1} - c_{t+1}^D R_{t+1}^f - [c_{t+1}^D (R_{t+1} - Q_{t+1} + Q_t) - c_{t+1}^D (R_{t+1} - Q_{t+1} + Q_t)] = 0$$

$$c_{t+1}^D R_{t+1} - c_{t+1}^D R_{t+1}^f - [c_{t+1}^D R_{t+1} - c_{t+1}^D R_{t+1}^f] = 0$$

$$E_t(c_{t+1}^D R_{t+1}^D) = 0$$

This results is the same as in GLR.

However, notice that there is no z in either expression:

Substitute expressions for c_{t+1}^D from Section 2.2 (i.e., $c_{t+1}^D = \frac{(1+\beta)(1-\beta)}{1-\beta G+\beta} z_{t+1}^D - \frac{G}{1-\beta G+\beta} c_{t+1}^D$)

and R_{t+1}^D from Section 2.8 (i.e., $R_{t+1}^D = \frac{(1-\beta)(1+\beta)(1-\beta)}{(1-\beta G+\beta)} z_{t+1}^D - \frac{(1-\beta)G(\beta+1)}{(1-\beta G+\beta)} c_{t+1}^D$)

Hence,

$$y_t = a_h \hat{R}P_t + \hat{Z}_t + \hat{L}_t + (1 - a_h) \hat{R}P_{t-1} + (1 - a_h) \hat{Z}_t + \hat{Z}_t + \hat{L}_t^i; \quad (4)$$

$$y_t = a_h \hat{R}P_t + \hat{Z}_t + (1 - a_h) \hat{Z}_t + \hat{L}_t + (1 - a_h) \hat{R}P_{t-1} + \hat{Z}_t + \hat{L}_t^i; \quad (5)$$

Next, take a population-weighted average of equations (4) and (5), and denote y_t^w as:

$$y_t^w = a_h y_t + (1 - a_h) y_t =$$

that this is the same system of equations as in GLR. It follows that the change in production structure and demand fulfillment from the one in GLR to the one we are studying in this paper matters for how world production is allocated between the two countries but not for the overall amount of world production.

References

Ghironi, F., Lee, J., & Rebucci, A. (2015). The valuation channel of external adjustments